

The dual parametrization for gluon GPDs

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Abstract

We consider the application of the dual parametrization for the case of gluon GPDs in the nucleon. This provides opportunities for the more flexible modeling unpolarized gluon GPDs in a nucleon which in particular contain the invaluable information on the fraction of nucleon spin carried by gluons. We perform the generalization of Abel transform tomography approach for the case of gluons. We also discuss the skewness effect in the framework of the dual parametrization. We strongly suggest to employ the fitting strategies based on the dual parametrization to extract the information on GPDs from the experimental data.

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I. INTRODUCTION

Generalized parton distributions (GPDs) [1, 2] are considered to be a promising tool to study the partonic structure of hadrons (see Refs. [3–6] for recent reviews). Hard exclusive reactions, which can be described in the theoretical framework of GPDs, provide us opportunity to access the information on GPDs experimentally.

The importance of GPDs was widely realized in connection with the possibility to study the total angular momentum of partons in the nucleon. This allows to address the fundamental question how the total nucleon spin is made up from the contributions due to quarks and gluons. The famous Ji sum rule [2] relates the appropriate Mellin moments of quark and gluon GPDs¹ $H^{q,g}(x, \xi, t)$ and $E^{q,g}(x, \xi, t)$ to the fractions $J^{q,g}$ of the total angular momentum (*i.e.* the sum of parton spin and orbital angular momentum) carried by quarks of the flavor q and gluons respectively:

$$\begin{aligned} \int_{-1}^1 dx x [H^q(x, \xi, t=0) + E^q(x, \xi, t=0)] &= 2J^q; \\ \int_0^1 dx [H^g(x, \xi, t=0) + E^g(x, \xi, t=0)] &= 2J^g. \end{aligned} \quad (1)$$

Constraining J^g from the experiment would add an important lacking “piece” to the tricky “nucleon spin puzzle”. This requires the knowledge of gluon GPDs H^g , E^g as functions of x for fixed values of ξ and t . These GPDs can be most preferably studied in the hard exclusive electroproduction of $J^{PC} = 1^{--}$ mesons (ρ^0, ω, ϕ). For these meson electroproduction channels H^g and E^g make contributions at leading order in $1/Q$ (Q^2 refers to the initial photon virtuality) and in α_s . Hard exclusive electroproduction of mesons and particularly the exclusive electroproduction of ρ^0 is now in focus of intensive experimental investigations (see *e.g.* [7]). The important set of data have been already published [8]. Future experiments will provide even more precise data over a broader phase space. Thus, further theoretical development of GPD formalism that would allow the interpretation of this new experimental information is highly demanded.

Unfortunately, the problem of GPDs extraction from the data is complicated by the fact that GPDs depend on several variables (longitudinal momentum fraction of partons x , skewness parameter ξ , momentum transfer squared t and the factorization scale). More-

¹ For the definition of nucleon GPDs we use the set of conventions employed in [4]. See below.

over, only the integral convolutions of GPDs with certain convolution kernels rather than GPD themselves enter the observable quantities. Thus, in order to extract the information on GPDs from the data, one of necessity has to rely upon different phenomenologically motivated parametrizations of GPDs and ingenious fitting procedures for the observable quantities. One of the most popular parametrizations of GPDs is the famous Radyushkin double distribution Ansatz (RDDA) [9] employed in numerous phenomenological applications. In particular, the specific version of RDDA was adopted for gluon GPDs in the nucleon [10, 11].

There is the growing confidence that the present inability to describe some aspects of the available experimental data on the hard exclusive processes may be due to incomplete or inexact way of modelling GPDs (see *e.g.* discussions in [12–14]). A possible alternative way to parameterize GPDs consists in employing of the so-called dual parametrization of GPDs [15]. In this paper we apply the dual parametrization approach, which was originally developed for quark GPDs, to the case of gluon GPDs in the nucleon. We also perform the generalization of Abel transform tomography method [16] for the case of gluons. Finally, we discuss some aspects of the fitting strategy for the hard exclusive processes observables based on the dual parametrization of GPDs.

II. BASIC DEFINITIONS

Following the conventions accepted in [4], the unpolarized gluon GPDs in nucleon H^g , E^g are defined as the Fourier transform of the matrix element of the nonlocal gluon operator between the nucleon states according to:

$$\begin{aligned} & \frac{1}{P \cdot n} \int \frac{d\lambda}{2\pi} e^{i\lambda x P \cdot n} \langle N(p') | F^{+\nu}(-\lambda n/2) F_{\nu}^{+}(\lambda n/2) | N(p) \rangle \\ &= \frac{1}{2P \cdot n} \bar{U}(P + \frac{\Delta}{2}) \left[H^g(x, \xi, t) n_{\mu} \gamma^{\mu} + \frac{1}{2m_N} E^g(x, \xi, t) i\sigma^{\mu\nu} n_{\mu} \Delta_{\nu} \right] U(P - \frac{\Delta}{2}). \end{aligned} \quad (2)$$

The polarized gluon GPDs in nucleon \tilde{H}^g , \tilde{E}^g are defined as

$$\begin{aligned} & \frac{-i}{P \cdot n} \int \frac{d\lambda}{2\pi} e^{i\lambda x P \cdot n} \langle N(p') | F^{+\nu}(-\lambda n/2) \tilde{F}_{\nu}^{+}(\lambda n/2) | N(p) \rangle \\ &= \frac{1}{2P \cdot n} \bar{U}(P + \frac{\Delta}{2}) \left[\tilde{H}^g(x, \xi, t) n_{\mu} \gamma^{\mu} \gamma^5 + \frac{1}{2m_N} \tilde{E}^g(x, \xi, t) \gamma_5 n_{\mu} \Delta^{\mu} \right] U(P - \frac{\Delta}{2}). \end{aligned} \quad (3)$$

In (2), (3) we employ the standard notations: n is the light-cone direction ($n^2 = 0$, $P \cdot n \equiv P^+$), $P = \frac{1}{2}(p + p')$, $\Delta = p' - p$, $t = \Delta^2$, the skewness variable ξ refers to

$\Delta^+ = -2\xi P^+$; $\tilde{F}^{\alpha\beta} \equiv \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$ is the dual gluon field strength. Throughout this paper we adopt the light-cone gauge $A^+ = 0$, so that the gauge link does not appear in the operators in definitions (2), (3).

The definition (2) differs from other definitions encountered in the literature (see [4]). For $0 \leq x \leq 1$:

$$\begin{aligned} H^g(x, \xi, t)|_{\text{here}} &= H^g(x, \xi, t)|_{[3]} + H^g(-x, \xi, t)|_{[3]} = x \left(H^g(x, \xi, t)|_{[17]} - H^g(-x, \xi, t)|_{[17]} \right) . \\ \tilde{H}^g(x, \xi, t)|_{\text{here}} &= \tilde{H}^g(x, \xi, t)|_{[3]} - \tilde{H}^g(-x, \xi, t)|_{[3]} = x \left(\tilde{H}^g(x, \xi, t)|_{[17]} + \tilde{H}^g(-x, \xi, t)|_{[17]} \right) . \end{aligned}$$

The same relations hold for E^g and \tilde{E}^g respectively.

As the gluon itself is its own antiparticle gluon GPDs $H^g(x, \xi, t)$, $E^g(x, \xi, t)$ defined in (2) are even functions of x :

$$H^g(x, \xi, t) = H^g(-x, \xi, t); \quad E^g(x, \xi, t) = E^g(-x, \xi, t). \quad (4)$$

Gluon GPDs $\tilde{H}^g(x, \xi, t)$, $\tilde{E}^g(x, \xi, t)$ defined in (3) are odd functions of x :

$$\tilde{H}^g(x, \xi, t) = -\tilde{H}^g(-x, \xi, t); \quad \tilde{E}^g(x, \xi, t) = -\tilde{E}^g(-x, \xi, t). \quad (5)$$

Let us stress that in what follows we consider the gluon GPDs in nucleon (2), (3) on the interval $0 \leq x \leq 1$.

In the forward limit gluon GPDs H^g and \tilde{H}^g reduce to usual forward gluon distributions in the nucleon $g(x)$ and $\Delta g(x)$, while GPD E^g and \tilde{E}^g are reduced to unknown gluon distributions, which we denote as $e^g(x)$ and $\Delta e^g(x)$:

$$\begin{aligned} H^g(x, 0, 0) &= xg(x); \quad E^g(x, 0, 0) = xe^g(x); \\ \tilde{H}^g(x, 0, 0) &= x\Delta g(x); \quad \tilde{E}^g(x, 0, 0) = x\Delta e^g(x). \end{aligned} \quad (6)$$

Note that the forward gluon distributions $g(x)$, $\Delta g(x)$ and $e^g(x)$, $\Delta e^g(x)$ are continued to the negative value of their argument according to:

$$\begin{aligned} g(x) &= -g(-x); \quad e^g(x) = -e^g(-x); \\ \Delta g(x) &= \Delta g(-x); \quad \Delta e^g(x) = \Delta e^g(-x). \end{aligned} \quad (7)$$

The Mellin moments in the momentum fraction x are of major importance in the GPD approach. According to the polynomiality property of GPDs the x Mellin moments of gluon GPDs defined in (2), (3) are polynomials of ξ . The coefficients of these polynomials are related to the form factors of the local twist two gluon operators:

$$\begin{aligned}\mathcal{O}_g^{\mu\mu_1\ldots\mu_n\nu} &= \mathbf{S} F^{\mu\alpha} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n} F_\alpha^\nu; \\ \tilde{\mathcal{O}}_g^{\mu\mu_1\ldots\mu_n\nu} &= \mathbf{S}(-i) F^{\mu\alpha} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n} \tilde{F}_\alpha^\nu.\end{aligned}\tag{8}$$

Here, as usual, D^μ is the covariant derivative, \mathbf{S} denotes symmetrization in all uncontracted Lorentz indices and subtraction of the appropriate traces. More technically, the polynomiality property for unpolarized gluon GPDs means that for odd N :

$$\begin{aligned}\int_0^1 dx x^{N-1} H^g(x, \xi, t) &= \sum_{\substack{k=0 \\ \text{even}}}^{N+\text{mod}(N,2)} \xi^k h_{N,k}^g(t); \\ \int_0^1 dx x^{N-1} E^g(x, \xi, t) &= \sum_{\substack{k=0 \\ \text{even}}}^{N+\text{mod}(N,2)} \xi^k e_{N,k}^g(t) \\ \text{with } e_{N,N+1}^g(t) &= -h_{N,N+1}^g(t).\end{aligned}\tag{9}$$

For the case of polarized gluon GPDs for even N

$$\begin{aligned}\int_0^1 dx x^{N-1} \tilde{H}^g(x, \xi, t) &= \sum_{\substack{k=0 \\ \text{even}}}^N \xi^k \tilde{h}_{N,k}^g(t); \\ \int_0^1 dx x^{N-1} \tilde{E}^g(x, \xi, t) &= \sum_{\substack{k=0 \\ \text{even}}}^N \xi^k \tilde{e}_{N,k}^g(t).\end{aligned}\tag{10}$$

III. THE DUAL PARAMETRIZATION FOR GLUON GPDS

Historically the first non-trivial phenomenological para-metrization of GPDs was the famous Radyushkin double distribution Ansatz (RDDA) suggested within the double distribution representation of GPDs [9]. In particular, a version of RDDA was adopted for the case of gluon GPD H^g in the nucleon (2) [10, 11]:

$$H^g(x, \xi, t=0) = H_{DD}^g(x, \xi) + 2\theta(\xi - |x|)\xi D^g\left(\frac{x}{\xi}\right),\tag{11}$$

where D^g stands for the gluon D -term [3] and H_{DD}^g is built as a one dimensional section of a two-variable double distribution:

$$H_{DD}^g(x, \xi) = \int_0^1 d\beta \int_{-1+\beta}^{1-\beta} d\alpha \{ \delta(x - \beta - \alpha\xi) - \delta(x + \beta - \alpha\xi) \} h^{(b)}(\beta, \alpha) \beta g(\beta). \quad (12)$$

The profile function $h^{(b)}(\beta, \alpha)$ is parameterized as

$$h^{(b)}(\beta, \alpha) = \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{[(1-|\beta|)^2 - \alpha^2]^b}{(1-|\beta|)^{2b+1}}. \quad (13)$$

The parameter b characterizes the strength of ξ dependence of the resulting GPD. The usual choice for the gluon case is $b = 2$. This is motivated by the interpretation of the α dependence like a meson distribution amplitude for hard exclusive processes. The cases $b = 2$ correspond to the asymptotic behavior of a gluon distribution amplitude $\sim (1 - \alpha^2)^2$.

The achieved theoretical understanding of both theoretical and experimental aspects of GPD physics hints (see *e.g.* [12, 13, 18]) at the necessity to introduce new GPD parametrizations which should be more general and flexible than the basic form of the RDDA employed in present-day mainstream phenomenology.

The possible alternative way to parameterize GPDs is the so-called dual parametrization [15] (see [16, 19–22] for the recent development and discussion). Originally, the dual parametrization was formulated for the case of quark GPDs. Now we are going to generalize this approach for the gluon case.

In the framework of the dual parametrization GPDs are presented as infinite sums of the t -channel² Regge exchanges [23]. Let us stress that the term “dual” is intended to lay emphasis on the natural association with the old idea of duality in hadron-hadron low energy scattering. The essence of the duality hypothesis for binary scattering amplitudes [24, 25] can be summarized as the assumption that the infinite sum over only the cross-channel Regge exchanges may provide the complete description of the whole scattering amplitude in a certain domain of kinematical variables.

More technically, in the dual parametrization the t -channel matrix element of the particular non-local light ray operator \hat{O} between the hadron states which enters the definition of

² The t -channel refers to the t -channel of the hard exclusive electroproduction reaction in question. *E.g.* for the case of DVCS this is hadron pair production $\gamma^*\gamma \rightarrow N\bar{N}$.

GPD (see *e.g.* (2), (3)) is presented in the following form:

$$\langle N(p')N(-p)|\hat{O}|0\rangle \sim \sum_{R_J} \sum_{\text{polarization of } R_J} \frac{1}{t - M_{R_J}^2} \times \underbrace{\langle N(p')N(-p)|R_J\rangle}_{R_J N \bar{N} \text{ effective vertex}} \underbrace{\langle R_J|\hat{O}|0\rangle}_{\text{F.T. of DA of } R_J}. \quad (14)$$

The sum in (14) stands over all possible t -channel meson resonance exchanges with suitable quantum numbers and of arbitrary high spin J and mass M_{R_J} . For the classification of $R_J N \bar{N}$ vertices see *e.g.* ref. [26]. The distribution amplitude (DA) of the spin- J t -channel resonance occurring in $\langle R_J|\hat{O}|0\rangle$ matrix element is expanded in the eigenfunctions of the ERBL (Efremov, Radyushkin, Brodsky, Lepage) evolution equation. For the gluon case these are the Gegenbauer polynomials $C_n^{\frac{5}{2}}(z)$ [27]. The on shell spin sum of spin- J t -channel resonance resulting from the sum over polarization in (14) is expanded in the t -channel partial waves.

Thus, in the framework of the dual parametrization GPDs have the form of double expansions in the eigenfunctions of the ERBL kernel and in the t -channel partial waves. These formal series are then analytically continued to the physical region by means of the cunningly organized summation procedure. It is worthy to mention that the dual parametrization of GPDs shares many common features with the expansion of GPDs in collinear conformal partial waves [28] known in different versions [12, 14, 29–32]. The expansion of GPDs in collinear conformal partial waves arises naturally from the solution of the evolution equations to the leading order accuracy. In both cases the angular momentum of SO(3) partial waves expansion in the t -channel partial waves as well as the conformal spin appear as labels.

The important point is that one has to take properly into account the complication introduced by the fact that the nucleon has spin- $\frac{1}{2}$. In order to be able to write down the formal series for the gluon GPDs in the nucleon in the framework of the dual parametrization it is necessary to point out the combinations of gluon GPDs H^g , \tilde{H}^g and E^g , \tilde{E}^g suitable for the partial wave expansion in the t -channel partial waves. This is an easy task since the Lorentz structure of the Fourier transform of nucleon matrix element of gluon light cone operators which enter the definitions (2), (3) is obviously the same as that of the quark light cone operator familiar from the definition of unpolarized and polarized quark GPDs respectively. Thus, in the complete analogy the case unpolarized quark GPDs [4, 33] the partial wave expansion in the t -channel partial waves can be written for the electric and

magnetic combinations of unpolarized gluon GPDs

$$\begin{aligned} H^{g(E)}(x, \xi, t) &= H^g(x, \xi, t) + \tau E^g(x, \xi, t); \quad (\tau \equiv \frac{t}{4m_N^2}) \\ H^{g(M)}(x, \xi, t) &= H^g(x, \xi, t) + E^g(x, \xi, t). \end{aligned} \quad (15)$$

and for the following combinations of polarized gluon GPDs:

$$\begin{aligned} \tilde{H}^g(x, \xi, t); \\ \tilde{H}^{g(PS)}(x, \xi, t) &= \tilde{H}^g(x, \xi, t) + \tau \tilde{E}^g(x, \xi, t). \end{aligned} \quad (16)$$

One may come to the same conclusion employing the general method for determining the invariant amplitudes of a binary scattering process suggested in [34, 35] (see also [25]). Using this method it is also straightforward to check that the electric combination $H^{g(E)}$ and pseudoscalar combination $\tilde{H}^{g(PS)}$ are to be expanded in the Legendre polynomials of $\cos \theta_t$ $P_l(\cos \theta_t)$ while the magnetic combination $H^{g(M)}$ and \tilde{H}^g are to be expanded in the derivatives of the Legendre polynomials $P'_l(\cos \theta_t)$. Note that to the leading order of $1/\mathcal{Q}$ expansion the t -channel scattering angle³ is expressed through the kinematical variables as

$$\cos \theta_t = \frac{1}{\xi \sqrt{1 - \frac{4m_N^2}{t}}} + O\left(\frac{1}{\mathcal{Q}^2}\right). \quad (17)$$

For the electric combination of gluon GPDs $H^{g(E)}$ $J^{PC} = 0^{++}, 2^{++}, \dots$ intermediate meson states (*e.g.* f_0, f_2) contribute, while for the magnetic combination $H^{g(M)}$ the contributions of $J^{PC} = 2^{++}, 4^{++}, \dots$ intermediate meson states are relevant. For \tilde{H}^g $J^{PC} = 1^{++}, 3^{++}, \dots$ intermediate meson states contribute and, finally, for $\tilde{H}^{g(PS)}$ $J^{PC} = 0^{-+}, 2^{-+}, \dots$ meson states contribute.

Taking into account all these considerations one can write down the following partial wave expansion for electric and magnetic combinations of unpolarized gluon GPDs in the nucleon⁴:

$$H^{g(E)}(x, \xi, t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(E)}(t) \theta\left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right)^2 C_{n-1}^{\frac{5}{2}}\left(\frac{x}{\xi}\right) \xi P_l\left(\frac{1}{\xi}\right); \quad (18)$$

³ θ_t is defined as the scattering angle in the center of mass frame of the t -channel of the hard exclusive electroproduction reaction (*e.g.* $\gamma\gamma^* \rightarrow N\bar{N}$ for the DVCS).

⁴ Note, that contrary to the case of quark GPDs in nucleon defined in [22] we do not introduce the factor “2” in the partial wave expansions for gluon GPDs.

$$H^{g(M)}(x, \xi, t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(M)}(t) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right)^2 C_{n-1}^{\frac{5}{2}} \left(\frac{x}{\xi}\right) P'_l \left(\frac{1}{\xi}\right). \quad (19)$$

The partial wave expansion for the polarized gluon GPDs in the nucleon reads

$$\tilde{H}^g(x, \xi, t) = \sum_{\substack{n=2 \\ \text{even}}}^{\infty} \sum_{\substack{l=1 \\ \text{odd}}}^{n+1} \tilde{B}_{nl}^g(t) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right)^2 C_{n-1}^{\frac{5}{2}} \left(\frac{x}{\xi}\right) \xi P'_l \left(\frac{1}{\xi}\right); \quad (20)$$

$$\tilde{H}^{g(PS)}(x, \xi, t) = \sum_{\substack{n=2 \\ \text{even}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^n \tilde{B}_{nl}^{g(PS)}(t) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right)^2 C_{n-1}^{\frac{5}{2}} \left(\frac{x}{\xi}\right) P_l \left(\frac{1}{\xi}\right). \quad (21)$$

In fact, the most non-committal way to understand the partial wave expansions (18), (19), (20), (21) is to consider them just as formal series which satisfy the fundamental polynomiality property of gluon GPDs (9), (10). The summation procedure [15] allowing to convert these formal series into rigorously defined expressions is reviewed in the Appendix A.

In what follows we are going to consider in details the properties of unpolarized gluon GPDs in the framework of the dual parametrization. The summary of results for the case of polarized gluon GPDs in the nucleon is presented in the Appendix B.

Let us first consider the electric combination. For odd N the $(N-1)$ th Mellin moment is indeed the polynomial of ξ of order $N+1$

$$\begin{aligned} \int_0^1 dx x^{N-1} H^{g(E)}(x, \xi, t) &= \sum_{\substack{k=0 \\ \text{even}}}^{N+1} \xi^k h_{N,k}^{g(E)}(t) \\ &= \xi^N \sum_{\substack{n=1 \\ \text{odd}}}^N \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(E)}(t) \xi P_l \left(\frac{1}{\xi}\right) \frac{n(1+n)(2+n)(3+n) \Gamma(\frac{5}{2}) \Gamma(N)}{9 \cdot 2^N \Gamma(1 + \frac{-n+N}{2}) \Gamma(\frac{7}{2} + \frac{-2+n+N}{2})}. \end{aligned} \quad (22)$$

The set of coefficients $h_{N,k}^{g(E)}(t)$ can be expressed through the generalized form factors $B_{nl}^{g(E)}$ according to

$$\begin{aligned} h_{N,k}^{g(E)}(t) &= \sum_{\substack{n=1 \\ \text{odd}}}^N \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(E)}(t) (-1)^{\frac{k+l-N-1}{2}} \frac{\Gamma(\frac{2-k+l+N}{2})}{3 \cdot 2^{k+1} \Gamma(\frac{1+k+l-N}{2}) \Gamma(2-k+N)} \\ &\times \frac{n(1+n)(2+n)(3+n) \Gamma(N)}{\Gamma(\frac{2-n+N}{2}) \Gamma(\frac{5+n+N}{2})}. \end{aligned} \quad (23)$$

For the magnetic combination the $(N - 1)$ th Mellin moment (N - odd) is the polynomial of ξ of order $N - 1$

$$\int_0^1 dx x^{N-1} H^{g(M)}(x, \xi, t) = \sum_{\substack{k=0 \\ \text{even}}}^{N-1} \xi^k h_{N,k}^{g(M)}(t) = \xi^N \sum_{\substack{n=1 \\ \text{odd}}}^N \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(M)}(t) \xi P'_l \left(\frac{1}{\xi} \right) \frac{n(1+n)(2+n)(3+n) \Gamma(\frac{5}{2}) \Gamma(N)}{9 \cdot 2^N \Gamma(1 + \frac{-n+N}{2}) \Gamma(\frac{7}{2} + \frac{-2+n+N}{2})}. \quad (24)$$

The corresponding set of coefficients $h_{N,k}^{g(M)}(t)$ is expressed through the generalized form factors $B_{nl}^{g(M)}$ as follows

$$h_{N,k}^{g(M)}(t) = \sum_{\substack{n=1 \\ \text{odd}}}^N \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(M)}(t) (-1)^{\frac{k+l-N+1}{2}} \frac{(-1+k-N) \Gamma(\frac{2-k+l+N}{2})}{3 \cdot 2^{k+1} \Gamma(\frac{1+k+l-N}{2}) \Gamma(2-k+N)} \times \frac{n(1+n)(2+n)(3+n) \Gamma(N)}{\Gamma(\frac{2-n+N}{2}) \Gamma(\frac{5+n+N}{2})}. \quad (25)$$

Gluon GPDs $H^{g(E,M)}(x, \xi, t)$ are normalized according to:

$$\int_0^1 dx H^{g(E)}(x, \xi, t) = M_2^g(t) + \frac{4}{5}(1 - \tau) d_1^g(t) \xi^2; \\ \int_0^1 dx H^{g(M)}(x, \xi, t) = 2J^g(t). \quad (26)$$

Here $M_2^g(t)$ stands for the t -dependent momentum fraction carried by gluons in the nucleon; $J^g(t)$ denotes the t -dependent fraction of angular momentum carried by gluons and $d_1^g(t)$ is the first coefficient of Gegenbauer expansion of the gluon D -term (52).

IV. THE PROPERTIES OF $H^{g(E,M)}(x, \xi, t)$ IN THE DUAL PARAMETRIZATION

To sum up the formal series for the electric and magnetic combinations of gluon GPDs (18), (19) we employ the techniques developed in [15] (see also discussion in [22]). Some of the additional technical details specific for the gluon case are presented in the Appendix A.

To proceed with the summation of the formal series (18), (19) we introduce two sets of gluon forward-like functions $G_{2\nu}^{g(E)}$ and $G_{2\nu}^{g(M)}$, whose Mellin moments generate electric and magnetic generalized form-factors $B_{nl}^{g(E,M)}(t)$:

$$B_{nn+1-2\nu}^{g(E,M)}(t) = \int_0^1 dx x^n G_{2\nu}^{g(E,M)}(x, t). \quad (27)$$

The explicit expression for the electric combination of gluon GPDs $H^{g(E)}$ through the corresponding forward like functions reads:

$$H^{g(E)}(x, \xi, t) = \sum_{\nu=0}^{\infty} \frac{\xi^{2\nu}}{2} [H^{g(E)(\nu)}(x, \xi, t) + H^{g(E)(\nu)}(-x, \xi, t)] \\ + \sum_{\nu=1}^{\infty} \theta\left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right)^2 \xi C_{2\nu-2}^{\frac{5}{2}}\left(\frac{x}{\xi}\right) B_{2\nu-1,0}^{g(E)}(t). \quad (28)$$

Note, that the second term in (28) is the pure D -term contribution. The result for the magnetic combination of gluon GPDs $H^{g(M)}$ can be obtained in the similar way applying the differential operator $\left(1 - x \frac{\partial}{\partial x} - \xi \frac{\partial}{\partial \xi}\right)$:

$$H^{g(M)}(x, \xi, t) = \sum_{\nu=0}^{\infty} \left[\left(1 - x \frac{\partial}{\partial x} - \xi \frac{\partial}{\partial \xi}\right) \frac{\xi^{2\nu}}{2} [H^{g(M)(\nu)}(x, \xi, t) + H^{g(M)(\nu)}(-x, \xi, t)] \right]. \quad (29)$$

The functions $H^{g(E,M)(\nu)}(x, \xi, t)$ which appear in (28) and (29) defined for $-\xi \leq x \leq 1$ are given by the following integral transformations:

$$H^{g(E,M)(\nu)}(x, \xi, t) \\ = \theta(x > \xi) \frac{1}{\pi} \int_{y_0}^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2}\right) G_{2\nu}^{(E,M)}(y, t) \right] \int_{s_1}^{s_2} ds \frac{x_s^{2-2\nu} (1 - s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \\ + \theta(|x| < \xi) \frac{1}{\pi} \int_0^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2}\right) G_{2\nu}^{(E,M)}(y, t) \right] \left\{ \int_{s_1}^{s_3} ds \frac{x_s^{2-2\nu} (1 - s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \right. \\ \left. - \frac{\pi}{\xi^{2\nu}} \left(1 - \frac{x^2}{\xi^2}\right)^2 \sum_{l=-1}^{2\nu-3} C_{2\nu-l-3}^{\frac{5}{2}}\left(\frac{x}{\xi}\right) \xi P_l\left(\frac{1}{\xi}\right) \frac{6y^{2\nu-l-2}}{(2\nu-l)(2\nu-l+1)} \right\}, \quad (30)$$

with $P_{-n}(\chi) \equiv P_{n-1}(\chi)$. Here, as usual, $x_s = 2 \frac{x-\xi s}{(1-s^2)y}$; s_i , ($i = 1, \dots, 4$) stand for the four roots of the equation $2x_s - x_s^2 - \xi^2 = 0$ (see (A12) for the definitions) and y_0 is defined in (A14). The integrals in (30) are well convergent for the set of the forward like functions with the following small- y behavior $G_{2\nu}(y) \sim \frac{1}{y^{2\nu+\alpha}}$ with $\alpha < 2$.

Point $\xi = x$

The important limiting case in which the expressions (28), (29) are reduced to much simpler forms is the point $x = \xi$.

$$\begin{aligned} H^{g(E)}(\xi, \xi, t) &= \frac{2}{3\pi} \xi \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^1 \frac{dy}{y} \sum_{\nu=0}^{\infty} y^{2\nu} G_{2\nu}^{(E)}(y, t) \left[\frac{1}{\sqrt{\frac{2y}{\xi} - y^2 - 1}} \right]; \\ H^{g(M)}(\xi, \xi, t) &= -\xi^2 \frac{\partial}{\partial \xi} \frac{2}{3\pi} \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^1 \frac{dy}{y} \sum_{\nu=0}^{\infty} y^{2\nu} G_{2\nu}^{(M)}(y, t) \left[\frac{1}{\sqrt{\frac{2y}{\xi} - y^2 - 1}} \right]. \end{aligned} \quad (31)$$

Point $\xi = 1$

Another notable limiting case is $\xi = 1$:

$$\begin{aligned} H^{g(E)}(x, \xi = 1, t) &= \frac{1}{2} (1 - x^2)^2 \sum_{\nu=0}^{\infty} \int_0^1 dy y \left[\frac{1}{(1 - 2xy + y^2)^{\frac{5}{2}}} + \frac{1}{(1 + 2xy + y^2)^{\frac{5}{2}}} \right. \\ &\quad \left. - 2 \sum_{\substack{j=0 \\ \text{even}}}^{2\nu-2} y^j C_j^{\frac{5}{2}}(x) \right] G_{2\nu}^{(E)}(y, t); \end{aligned} \quad (32)$$

and

$$\begin{aligned} H^{g(M)}(x, \xi = 1, t) &= \frac{1}{2} (1 - x^2)^2 \sum_{\nu=0}^{\infty} \int_0^1 dy \left[\frac{1}{(1 - 2xy + y^2)^{\frac{5}{2}}} + \frac{1}{(1 + 2xy + y^2)^{\frac{5}{2}}} \right. \\ &\quad \left. - 2 \sum_{\substack{j=0 \\ \text{even}}}^{2\nu-2} y^j C_j^{\frac{5}{2}}(x) \right] y^{3-2\nu} \frac{\partial}{\partial y} \left(y^{2\nu} \frac{\partial}{\partial y} G_{2\nu}^{(M)}(y, t) \right). \end{aligned} \quad (33)$$

Forward limit and $G_0^{(E,M)}(x, t)$

Let us introduce the convenient notations for the combinations of the t -dependent parton densities to which GPDs $H^{g(E,M)}$ are reduced in the limit $\xi \rightarrow 0$:

$$\begin{aligned} H^{g(E)}(x, \xi = 0, t) &= xg(x, t) + \tau x e^g(x, t) \equiv xg^{(E)}(x, t); \\ H^{g(M)}(x, \xi = 0, t) &= xg(x, t) + x e^g(x, t) \equiv xg^{(M)}(x, t), \end{aligned} \quad (34)$$

where $g(x, t)$ and $e^g(x, t)$ stands for the t -dependent gluon distributions in the nucleon ($xe^g(x, t) \equiv E^g(x, \xi = 0, t)$). Employing the general results (23), (25) one can check that for odd N :

$$\begin{aligned} \int_0^1 dx x^{N-1} H^{g(E)}(x, \xi = 0, t) &= \int_0^1 dx x^N g^{(E)}(x, t) = \frac{(N+2)(N+3)}{3(2N+3)} B_{N, N+1}^{g(E)}(t); \\ \int_0^1 dx x^{N-1} H^{g(M)}(x, \xi = 0, t) &= \int_0^1 dx x^N g^{(M)}(x, t) = \frac{(N+1)(N+2)(N+3)}{3(2N+3)} B_{N, N+1}^{g(M)}(t). \end{aligned} \quad (35)$$

Inverting the Mellin moments in (35) we express the forward like functions $G_0^{(E, M)}$ related to the t -dependent gluon distributions

$$\begin{aligned} G_0^{(E)}(x, t) &= 9x^2 \int_x^1 \frac{dy}{y^3} g^{(E)}(y, t) - 3x \int_x^1 \frac{dy}{y^2} g^{(E)}(y, t); \\ G_0^{(M)}(x, t) &= -\frac{9}{2}x^2 \int_x^1 \frac{dy}{y^3} g^{(M)}(y, t) + 3x \int_x^1 \frac{dy}{y^2} g^{(M)}(y, t) + \frac{3}{2} \int_x^1 \frac{dy}{y} g^{(M)}(y, t). \end{aligned} \quad (36)$$

The normalization is

$$\begin{aligned} \int_0^1 dx x G_0^{(E)}(x, t) &= \frac{5}{4} M_2^g(t); \\ \int_0^1 dx x G_0^{(M)}(x, t) &= \frac{5}{8} 2J^g(t). \end{aligned} \quad (37)$$

Thus the information on the fraction of nucleon total angular momentum carried by gluons is encoded in the magnetic forward like function $G_0^{(M)}$.

Small ξ expansion

It is helpful to consider the expansion of electric and magnetic combinations of nucleon gluon GPDs in powers of ξ around $\xi = 0$ for fixed x ($x > \xi$). For the electric combination

$H^{g(E)}$ the corresponding expansion to the order ξ^2 it is given by:

$$\begin{aligned}
H^{g(E)}(x, \xi, t) = & \frac{5}{12}xG_0^{(E)}(x, t) - \frac{1}{6}x^2\frac{\partial}{\partial x}G_0^{(E)}(x, t) + \frac{1}{8}\int_x^1 dy \left(\frac{x}{y}\right)^{\frac{3}{2}} G_0^{(E)}(y, t) \\
& + \xi^2 \left[\frac{1}{32} \left(-\frac{7}{3x} + x \right) G_0^{(E)}(x, t) + \frac{1}{24}(1+x^2)\frac{\partial}{\partial x}G_0^{(E)}(x, t) + \frac{1}{24}x(1-x^2)\frac{\partial^2}{\partial x^2}G_0^{(E)}(x, t) \right. \\
& + \int_x^1 dy G_0^{(E)}(y, t) \left(\frac{5}{128} \left(\frac{x}{y}\right)^{\frac{1}{2}} + \frac{3}{128} \left(\frac{x}{y}\right)^{\frac{3}{2}} + \frac{1}{y^2} \left(\frac{3}{128} \left(\frac{y}{x}\right)^{\frac{1}{2}} + \frac{5}{128} \left(\frac{x}{y}\right)^{\frac{1}{2}} \right) \right) \\
& + \frac{7}{48}xG_2^{(E)}(x, t) - \frac{1}{24}x^2\frac{\partial}{\partial x}G_2^{(E)}(x, t) + \\
& \left. \int_x^1 dy G_2^{(E)}(y, t) \left(\frac{35}{256} \left(\frac{y}{x}\right)^{\frac{1}{2}} + \frac{5}{128} \left(\frac{x}{y}\right)^{\frac{1}{2}} + \frac{3}{256} \left(\frac{x}{y}\right)^{\frac{3}{2}} \right) \right] + O(\xi^4). \tag{38}
\end{aligned}$$

For the magnetic combination $H^{g(E)}$ the expansion to the order ξ^2 reads

$$\begin{aligned}
H^{g(M)}(x, \xi, t) = & \frac{1}{8}xG_0^{(M)}(x, t) - \frac{1}{4}x^2\frac{\partial}{\partial x}G_0^{(M)}(x, t) + \frac{1}{6}x^3\frac{\partial^2}{\partial x^2}G_0^{(M)}(x, t) \\
& - \frac{1}{16}\int_x^1 dy \left(\frac{x}{y}\right)^{\frac{3}{2}} G_0^{(M)}(y, t) + \xi^2 \left[\frac{1}{16x}G_0^{(M)}(x, t) + \frac{1}{32}(1-5x^2)\frac{\partial}{\partial x}G_0^{(M)}(x, t) \right. \\
& - \frac{1}{8}x(1-x^2)\frac{\partial^2}{\partial x^2}G_0^{(M)}(x, t) - \frac{1}{24}x^2(1-x^2)\frac{\partial^3}{\partial x^3}G_0^{(M)}(x, t) \\
& - \int_x^1 dy G_0^{(M)}(y, t) \left(\frac{15}{256} \left(\frac{x}{y}\right)^{\frac{1}{2}} + \frac{15}{256} \left(\frac{x}{y}\right)^{\frac{3}{2}} + \frac{1}{y^2} \left(\frac{3}{256} \left(\frac{y}{x}\right)^{\frac{1}{2}} + \frac{15}{256} \left(\frac{x}{y}\right)^{\frac{1}{2}} \right) \right) \\
& - \frac{5}{48}xG_2^{(M)}(x, t) - \frac{1}{48}x^2\frac{\partial}{\partial x}G_2^{(M)}(x, t) + \frac{1}{24}x^3\frac{\partial^2}{\partial x^2}G_2^{(M)}(x, t) \\
& \left. - \int_x^1 dy G_2^{(M)}(y, t) \left(\frac{35}{512} \left(\frac{y}{x}\right)^{\frac{1}{2}} + \frac{15}{256} \left(\frac{x}{y}\right)^{\frac{1}{2}} + \frac{15}{512} \left(\frac{x}{y}\right)^{\frac{3}{2}} \right) \right] + O(\xi^4). \tag{39}
\end{aligned}$$

Modelling $H^{g(E)}(x, \xi, t=0)$

On fig 1 we show the results of the numerical computation of $G_0^{(E)}$ contribution into the electric combination of gluon GPDs $H^{g(E)}$ at $t=0$ for several different values of ξ . As the numerical input for forward gluon distributions we used MRST LO fit [36] for nucleon parton distributions at $Q^2 = 1 \text{ GeV}^2$.

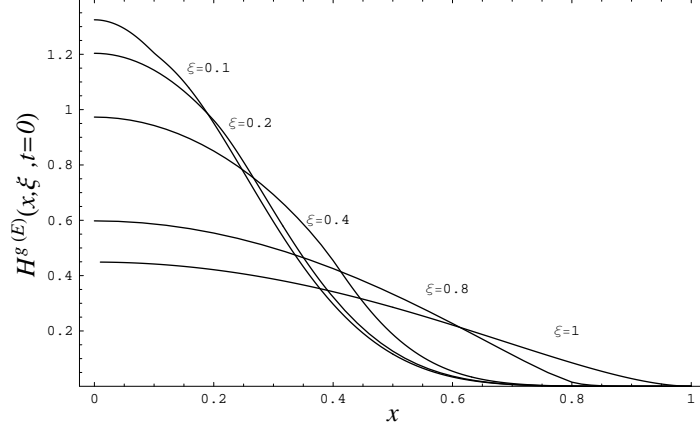


FIG. 1: Electric combinations of nucleon gluon GPDs $H^g(E)$, at $t = 0$ for different values of ξ . As the numerical input for forward gluon distributions we use MRST LO fit [36] for nucleon parton distributions at $Q^2 = 1 \text{ GeV}^2$.

V. ELEMENTARY GLUON AMPLITUDES AND ABEL TRANSFORM TOMOGRAPHY

The typical convolution integral involving gluon GPDs $F^g = \{H^g, E^g\}$ relevant for the calculation of hard exclusive $J^{PC} = 1^{--}$ meson electroproduction at the leading order in $1/Q$ and α_s reads [4]:

$$A^g(\xi, t) = \int_0^1 dx \frac{F^g(x, \xi, t)}{x} \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right]. \quad (40)$$

We define electric and magnetic elementary amplitudes

$$A^{g(E,M)}(\xi, t) = \int_0^1 dx \frac{H^{g(E,M)}(x, \xi, t)}{x} \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right]. \quad (41)$$

In order to obtain the partial wave expansions of the elementary amplitudes in the t -channel partial wave one has to substitute the formal series (18), (19) into the corresponding convolution integrals. For odd $n \geq 1$

$$\int_0^\xi dx \left(1 - \frac{x^2}{\xi^2}\right)^2 C_{n-1}^{\frac{5}{2}}\left(\frac{x}{\xi}\right) \frac{\xi}{x} \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] = \frac{4}{3}. \quad (42)$$

Thus

$$\begin{aligned}
A^{g(E)}(\xi, t) &= \frac{4}{3} \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(E)} P_l \left(\frac{1}{\xi} \right) ; \\
A^{g(M)}(\xi, t) &= \frac{4}{3} \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^{g(M)} \frac{1}{\xi} P'_l \left(\frac{1}{\xi} \right) .
\end{aligned} \tag{43}$$

These formal series can be summed exactly as for the case of singlet electric and magnetic quark GPDs. The form of the resulting expression actually differs only by a factor $\frac{1}{3}$. The expressions for gluon electric and magnetic elementary amplitudes read

$$\begin{aligned}
&A^{g(E)}(\xi, t) \\
&= \frac{2}{3} \int_0^1 \frac{dx}{x} \sum_{\nu=0}^{\infty} x^{2\nu} G_{2\nu}^{(E)}(x, t) \left[\frac{1}{\sqrt{1 - \frac{2x}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - 2\delta_{\nu 0} \right] ;
\end{aligned} \tag{44}$$

$$\begin{aligned}
&A^{g(M)}(\xi, t) \\
&= \left(-\xi \frac{\partial}{\partial \xi} \right) \frac{2}{3} \int_0^1 \frac{dx}{x} \sum_{\nu=0}^{\infty} x^{2\nu} G_{2\nu}^{g(M)}(x, t) \left[\frac{1}{\sqrt{1 - \frac{2x}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - 2\delta_{\nu 0} \right] .
\end{aligned} \tag{45}$$

We introduce electric and magnetic gluon GPD quint-essence functions:

$$\begin{aligned}
N^{g(E)}(x, t) &= \sum_{\nu=0}^{\infty} x^{2\nu} G_{2\nu}^{(E)}(x, t) ; \\
N^{g(M)}(x, t) &= \sum_{\nu=0}^{\infty} x^{2\nu} G_{2\nu}^{(M)}(x, t) .
\end{aligned} \tag{46}$$

The imaginary parts of gluon electric and magnetic elementary amplitudes then read:

$$\text{Im} A^{g(E)}(\xi, t) = \frac{\pi H^{g(E)}(\xi, \xi, t)}{\xi} = \frac{2}{3} \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^1 \frac{dx}{x} N^{g(E)}(x, t) \left[\frac{1}{\sqrt{\frac{2x}{\xi} - x^2 - 1}} \right] ; \tag{47}$$

$$\text{Im} A^{g(M)}(\xi, t) = \frac{\pi H^{g(M)}(\xi, \xi, t)}{\xi} = -\frac{2}{3} \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^1 dx \left\{ \frac{\partial}{\partial x} \frac{N^{g(M)}(x, t)}{1 - \xi x} \right\} \left[\frac{1}{\sqrt{\frac{2x}{\xi} - x^2 - 1}} \right] ; \tag{48}$$

The real part of the elementary electric gluon amplitude is given by

$$\begin{aligned} \text{Re}A^{g(E)}(\xi, t) &= \frac{2}{3} \int_0^{\frac{1-\sqrt{1-\xi^2}}{\xi}} \frac{dx}{x} N^{g(E)}(x, t) \\ &\times \left[\frac{1}{\sqrt{1 - \frac{2x}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - \frac{2}{\sqrt{1 + x^2}} \right] \\ &+ \frac{2}{3} \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^1 \frac{dx}{x} N^{g(E)}(x, t) \left[\frac{1}{\sqrt{1 + \frac{2x}{\xi} + x^2}} - \frac{2}{\sqrt{1 + x^2}} \right] + 2(1 - \tau)D^g(t), \end{aligned} \quad (49)$$

where $D^g(t)$ stands for the gluon D - form factor (51). Finally, the real part of the magnetic gluon amplitude reads:

$$\begin{aligned} \text{Re}A^{g(M)}(\xi, t) &= -\frac{2}{3} \int_0^{\frac{1-\sqrt{1-\xi^2}}{\xi}} dx \sqrt{1 + x^2} \left[\frac{1}{\sqrt{1 + x^2 - \frac{2x}{\xi}}} + \frac{1}{\sqrt{1 + x^2 + \frac{2x}{\xi}}} - \frac{2}{\sqrt{1 + x^2}} \right] \\ &\times \frac{\partial}{\partial x} \left(\frac{\sqrt{1 + x^2}}{1 - x^2} N^{g(M)}(x, t) \right) \\ &- \frac{2}{3} \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^1 dx \sqrt{1 + x^2} \left[\frac{1}{\sqrt{1 + x^2 - \frac{2x}{\xi}}} - \frac{2}{\sqrt{1 + x^2}} \right] \frac{\partial}{\partial x} \left(\frac{\sqrt{1 + x^2}}{1 - x^2} N^{g(M)}(x, t) \right). \end{aligned} \quad (50)$$

The gluon D -form factor is defined according to

$$D^g(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} d_n^g(t) = \int_{-1}^1 dz \frac{1}{1 - z^2} D^g(z, t), \quad (51)$$

where $D^g(z, t)$ stand for the gluon D -term for which the following Gegenbauer expansion was adopted in [3]:

$$D^g(z, t) = \frac{3}{4} (1 - z^2)^2 \left[d_1^g(t) + d_3^g(t) C_2^{\frac{5}{2}}(z) + d_5^g(t) C_4^{\frac{5}{2}}(z) + \dots \right]. \quad (52)$$

According to the analysis presented in [33, 37], the real part of the elementary amplitude $A^{g(E)}$ can be expressed through its imaginary part with the help a dispersion relation with one subtraction in the variable $\omega = \frac{1}{\xi}$ for the fixed value of t :

$$A^{g(E)}(\xi, t) = 2D^g(t) + \frac{1}{\pi} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi' - i\epsilon} - \frac{1}{\xi + \xi' - i\epsilon} \right) \text{Im}A^{g(E)}(\xi' - i\epsilon, t). \quad (53)$$

The subtraction constant is given by the value of the amplitude at the non-physical point $\omega = 0$ ($\xi = \infty$). It is known to be fixed by the D -term and equals $2D^g(t)$.

In [12, 13, 32] it was suggested to fix the subtraction constant in the dispersion relation (53) in terms of the imaginary part of the corresponding elementary amplitude assuming special analytical properties in j of the function

$$\Phi(j) \equiv \sum_{\nu=0}^{\infty} h_{2\nu+j, 2\nu}^{g(E)}(t) = \int_0^1 dx x^j \frac{1}{x} [H^{g(E)}(x, x, t) - H^{g(E)}(x, 0, t)] . \quad (54)$$

For entire odd $j \geq 1$ (54) defines in the family of sum rules for the specific combinations of coefficients $h_{2\nu+j, 2\nu}^{g(E)}$ at powers of ξ of the Mellin moments of GPD $H^{g(E)}$ (see eq. (22) for the definition). For $j = -1$ the integral is divergent since it is usually assumed that $\frac{1}{x}(H^{g(E)}(x, x, t) - H^{g(E)}(x, 0, t)) \sim 1/x^\alpha$ with $\alpha \sim 1$. In order to provide the desired expression for the D -form factor (54) is to be properly analytically continued to $j = -1$. To make it explicitly, let us assume that $\frac{1}{x}(H^{g(E)}(x, x, t) - H^{g(E)}(x, 0, t))$ belong to the class of functions with power like behavior for $x \sim 0$, which can be presented as the *finite* sums of singular terms [40]:

$$F : F(x) = \sum_{r=1}^R \frac{1}{x^{\alpha_r}} f_r(x) . \quad (55)$$

We suppose that for all $r = 1, \dots, R$ $\alpha_r < 2$ and $f_r(x)$ are arbitrary functions of x infinitely differentiable in the vicinity of $x = 0$. It is also supposed that $f_r(x)$ have zeroes of a sufficiently high order for $x = 1$. Then the relation for the gluon D form factor reads as in [12]:

$$2D^g(t) = \sum_{\nu=0}^{\infty} h_{2\nu-1, 2\nu}^{g(E)}(t) = \int_{(0)}^1 dx \frac{1}{x} \cdot \frac{1}{x} [H^{g(E)}(x, x, t) - H^{g(E)}(x, 0, t)] . \quad (56)$$

The lower integration limit “(0)” in (56) symbolize that we use the so-called analytic (or canonical) regularization [40]:

$$\int_{(0)}^1 dx \frac{f(x)}{x^{1+\alpha}} = \int_0^1 dx \frac{1}{x^{1+\alpha}} [f(x) - f(0) - xf'(0)] - f(0) \frac{1}{\alpha} - f'(0) \frac{1}{\alpha - 1} \quad (\text{for } \alpha < 2) . \quad (57)$$

In the framework of the dual parametrization assuming that $N^{g(E)}(x, t)$ and $G_0^{(E)}(x, t)$ also belong to class (55) (see discussion in [22]) one may rewrite the sum rule (56) for the

gluon D -form factor as

$$\begin{aligned}
D^g(t) &= \frac{2}{3} \int_0^1 \frac{dx}{x} G_0^{(E)}(x, t) \left(\frac{1}{\sqrt{1+x^2}} - 1 \right) + \frac{2}{3} \int_{(0)}^1 \frac{dx}{x} \left[N^{g(E)}(x, t) - G_0^{(E)}(x, t) \right] \frac{1}{\sqrt{1+x^2}}.
\end{aligned} \tag{58}$$

Thus the maximal amount of information on gluon GPDs which can be extracted from the leading order gluon amplitude of hard exclusive electroproduction of $J^{PC} = 1^{--}$ mesons can be quantified in terms of gluon GPD quintessence functions $N^{g(E, M)}$ and the value of gluon D -form factor D^g or, equivalently, in terms of $N^{g(E, M)}$ and $G_0^{(E)}$. The method of the Abel transform tomography for the dual parametrization of GPDs suggested in [16] allows to invert the integral convolutions for the imaginary parts of the elementary amplitudes in order to recover GPD quintessence functions. The generalization of this method for the case of gluon GPDs is straightforward. The expressions for gluon electric and magnetic GPD quintessence functions through the imaginary parts of the corresponding elementary amplitudes (41) read:

$$\begin{aligned}
N^{g(E)}(x, t) &= -\frac{3}{2\pi} \frac{x(1-x^2)}{(1+x^2)^{\frac{3}{2}}} \int_{\frac{2x}{1+x^2}}^1 d\xi \frac{1}{(\xi - \frac{2x}{1+x^2})^{\frac{3}{2}}} \left\{ \frac{1}{\sqrt{\xi}} \text{Im} A^{g(E)}(\xi, t) - \sqrt{\frac{1+x^2}{2x}} \text{Im} A^{g(E)}\left(\frac{2x}{1+x^2}, t\right) \right\} \\
&+ \frac{3}{2\pi} \frac{\sqrt{2x}(1+x)}{\sqrt{1+x^2}} \text{Im} A^{g(E)}\left(\frac{2x}{1+x^2}, t\right);
\end{aligned} \tag{59}$$

$$N^{g(M)}(x, t) = \frac{3}{2\pi} \frac{1-x^2}{\sqrt{1+x^2}} \int_{\frac{2x}{1+x^2}}^1 \frac{d\xi}{\sqrt{\xi}} \frac{1}{\sqrt{\xi - \frac{2x}{1+x^2}}} \text{Im} A^{g(M)}(\xi, t). \tag{60}$$

It is also extremely instructive to consider the small- ξ asymptotic behavior of $\text{Im} A^{g(E, M)}(\xi)$ in the framework of the dual parametrization. Assuming the power law behavior $N^{(E, M)}(x) \sim \frac{1}{x^\alpha}$ for gluon electric and magnetic GPD quintessence functions for small x we obtain for $\xi \sim 0$:

$$\begin{aligned}
\text{Im} A^{g(E)}(\xi) &\sim \frac{2^{\alpha+1}}{\xi^\alpha} \frac{\Gamma(\frac{1}{2})\Gamma(\alpha + \frac{1}{2})}{3\Gamma(\alpha + 1)}; \\
\text{Im} A^{g(M)}(\xi) &\sim \frac{2^{\alpha+1}}{\xi^\alpha} \frac{\Gamma(\frac{1}{2})\Gamma(\alpha + \frac{1}{2})}{3\Gamma(\alpha)}.
\end{aligned} \tag{61}$$

The case of particular importance is the so-called minimalist dual model in which only the contributions of the forward like functions $G_0^{(E, M)}(x)$ (36) are taken into account. Let

us assume the power law small- x asymptotic behavior for the electric and magnetic gluon densities

$$g^{(E,M)}(x) \sim \frac{1}{x^\alpha}. \quad (62)$$

For the forward like functions $G_0^{(E,M)}(x)$ the power law behavior (62) results in

$$\begin{aligned} G_0^{(E)}(x) &\sim \frac{3(1+2\alpha)}{(\alpha+1)(\alpha+2)} \frac{1}{x^\alpha}; \\ G_0^{(M)}(x) &\sim \frac{3(1+2\alpha)}{\alpha(\alpha+1)(\alpha+2)} \frac{1}{x^\alpha}. \end{aligned} \quad (63)$$

The contributions of the forward like functions $G_0^{(E,M)}(x)$ into $\text{Im}A^{g(E,M)}$ have the following asymptotic behavior for $\xi \sim 0$:

$$\text{Im}A_{G_0}^{g(E,M)}(\xi) \sim \frac{2^{\alpha+2}\Gamma(\frac{1}{2})\Gamma(\frac{3}{2}+\alpha)}{\Gamma(3+\alpha)} \frac{1}{\xi^\alpha}. \quad (64)$$

VI. SKEWNESS EFFECT FOR SMALL ξ IN THE DUAL PARAMETRIZATION

In this section we discuss some aspects of GPD modelling in the framework of the dual parametrization. The important characteristics of the particular GPD model is the so-called skewness effect (also known as the skewness ratio). In [13, 14] it was suggested to define the characteristics of the skewness effect for quark and gluon GPDs $H^{q,g}$ for $t = 0$ as ratios of the GPDs on the cross-over line $x = \xi$ to the appropriate parton distribution functions, to which GPDs are reduced in the forward limit. Thus, for quark GPD H^q the skewness ratio is defined according to

$$r^q = \frac{H^q(x = \xi, \xi, t = 0)}{H^q(\xi, 0, t = 0)} = \frac{H^q(\xi, \xi, 0)}{q(\xi)}, \quad (65)$$

while for gluon GPD H^g it reads

$$r^g = \frac{H^g(x = \xi, \xi, t = 0)}{H^g(\xi, 0, t = 0)} = \frac{H^g(\xi, \xi, 0)}{\xi g(\xi)}. \quad (66)$$

It is usually assumed that the small- ξ the asymptotic behavior of quark GPD on the cross over trajectory and the corresponding PDF is governed by the same leading Regge

trajectory:

$$H^q(\xi, \xi, t=0) = \frac{1}{\pi} \text{Im} A^q(\xi, t=0), \quad q(\xi) \sim \frac{1}{\xi^{a^q(t=0)}}. \quad (67)$$

Similarly, for the gluon case:

$$\frac{1}{\xi} H^g(\xi, \xi, t=0) = \frac{1}{\pi} \text{Im} A^g(\xi, t=0), \quad g(\xi) \sim \frac{1}{\xi^{a^g(t=0)}}. \quad (68)$$

For $\xi \sim 0$ this makes skewness ratios (65), (66) independent of ξ . The physical value of $a^{q,g}(t=0)$ in the relevant kinematical region is given by: $a^{q,g}(t=0, Q^2 = 4 \text{ GeV}^2) \equiv \alpha^{q,g} = 1.1 \div 1.2$. Obviously, the model without skewness in which $H^q(x, \xi) = q(x)$ and $H^g(x, \xi) = xg(x)$ corresponds to $r^{q,g} = 1$.

Quark skewness ratio can be directly related to the observable quantities. Since this point for some time was a rather knotty problem in the literature we present certain details of the relevant calculations following Ref. [14]. It is worth to mention that often in the literature somewhat different quantities are employed to indicate skewness effect at small ξ . They are defined as the following ratios [4]:

$$R^q = \frac{H^q(\xi, \xi, 0)}{q(2\xi)}; \quad R^g = \frac{H^g(\xi, \xi, 0)}{2\xi g(2\xi)}. \quad (69)$$

The relation between the two definitions of skewness effect under the assumption of Regge like asymptotic behavior (67), (68) is given by

$$R^q = 2^{\alpha^q} r^q; \quad R^g = 2^{\alpha^g-1} r^g. \quad (70)$$

The definition (69) is inspired by the fact that the observable ratio of DVCS and DIS cross sections is reduced to R^q rather than r^q . Let us present some details concerning this issue. In Ref. [41] in order to characterize the magnitude of skewness effects present in the DVCS process for small x_{Bj} the following ratio R was introduced as observable quantity:

$$R(Q^2, W) = \frac{4\sqrt{\pi\sigma_{DVCS} b(Q^2)}}{\sigma_T(\gamma^*p \rightarrow X)\sqrt{1+\rho^2}}, \quad (71)$$

where W stands for the total invariant energy, $b(Q^2)$ is the fitted t -slope parameter ($d\sigma_{DVCS}/dt \sim e^{-b|t|}$). In the LO analysis the DVCS cross section σ_{DVCS} in the small

x_{Bj} regime is governed by quark exchange mechanism and is known to be dominated by the contribution due to the imaginary part of the amplitude:

$$\sigma_{DVCS}(x_{Bj} \sim 0, Q^2) \simeq \frac{\alpha_{e.m.}^2 \pi x_{Bj}^2}{Q^4} |\text{Im} A_{DVCS}(\xi, t=0)|^2 \frac{1}{b(Q^2)} (1 + \rho^2). \quad (72)$$

The imaginary part of the leading order DVCS amplitude is given by $\text{Im} A_{DVCS}(\xi, t=0) = \pi H_{DVCS}(\xi, \xi, t=0)$, where H_{DVCS} is the combination of the singlet unpolarized quark GPDs $H_{DVCS} = \frac{4}{9}H^u + \frac{1}{9}H^d$. The parameter ρ^2 in (71), (72) refers to the small correction due to the real part of the amplitude. Since the virtual photon is assumed to be transversely polarized, in the case of DVCS it is also to be taken transversely polarized in the DIS amplitude in (71). The text book expression for the transversely polarized DIS cross section reads:

$$\sigma_T(\gamma^* p \rightarrow X) \simeq 4\pi^2 \alpha_{e.m.} \frac{F_2(x_{Bj}, Q^2)}{Q^2} = 4\pi^2 \alpha_{e.m.} \frac{x_{Bj} H_{DVCS}(x_{Bj}, \xi=0)}{Q^2}. \quad (73)$$

Now, since for small x_{Bj} the skewness parameter is given by $\xi \simeq x_{Bj}/2$, the definition (71) for the observable R can be rewritten as

$$R(Q^2, W)_{x_{Bj} \sim 0} \simeq \frac{H_{DVCS}(\frac{x_{Bj}}{2}, \frac{x_{Bj}}{2}, t=0)}{H_{DVCS}(x_{Bj}, \xi=0, t=0)}. \quad (74)$$

Contrary to the original statement of Ref. [41], the model without skewness corresponds to $R = 2^{\alpha^q} r^q \simeq 2$. Thus, as it is emphasized in [13, 14], the results of the experimental measurements of R performed by H1 Collaboration [41] presented on Figure 2 unambiguously allude to no skewness effect for small x_{Bj} .

Let us now discuss skewness effect in the framework of the dual parametrization. Employing the results of the previous section and of [9] we compute the skewness effect for small ξ in the minimalist dual model in which only the contributions of the forward like functions $Q_0^{(E,M)}(x)$ and $G_0^{(E,M)}(x)$ assuming Regge like behavior (67), (68) of input electric and magnetic combinations of parton densities. The corresponding result reads:

$$r_{Q_0}^{q(E,M)} \equiv \frac{H^q(E,M)(\xi, \xi)}{H^q(E,M)(\xi, 0)} \Big|_{\xi \sim 0} \simeq \frac{2^{\alpha^q} \Gamma(\alpha^q + \frac{3}{2})}{\Gamma(\frac{3}{2}) \Gamma(2 + \alpha^q)} \approx 3/2 \quad \text{for } \alpha^q \approx 1; \quad (75)$$

$$r_{G_0}^{g(E,M)} \equiv \frac{H^g(E,M)(\xi, \xi)}{H^g(E,M)(\xi, 0)} \Big|_{\xi \sim 0} \simeq \frac{2^{\alpha^g+1} \Gamma(\alpha^g + \frac{3}{2})}{\Gamma(\frac{3}{2}) \Gamma(3 + \alpha^g)} \approx 1 \quad \text{for } \alpha^g \approx 1. \quad (76)$$

It is extremely instructive to compare the skewness effect (76) in the minimalist dual model to that in the commonly used version of RDDA with the same asymptotic behavior

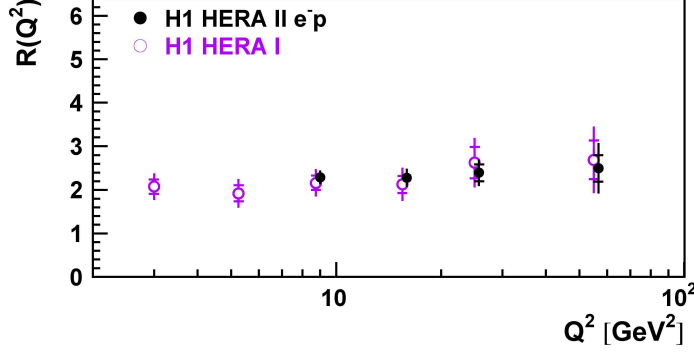


FIG. 2: The observable ratio R (71), shown as a function of Q^2 for fixed $W = 82$ GeV. This Figure is taken from [41].

(67), (68) of the input parton distributions. The result for the corresponding skewness ratio presented *e.g.* in [14] reads:

$$\begin{aligned}
 r_{RDA}^q &\equiv \frac{H_{DD}^q(\xi, \xi)}{H_{DD}^q(\xi, 0)} \simeq \frac{2^{2b-\alpha^q} \Gamma(b + \frac{3}{2}) \Gamma(1 + b - \alpha^q)}{\Gamma(\frac{3}{2}) \Gamma(2 + 2b - \alpha^q)}; \\
 r_{RDA}^g &\equiv \frac{H_{DD}^g(\xi, \xi)}{H_{DD}^g(\xi, 0)} \simeq \frac{2^{2b+1-\alpha^g} \Gamma(b + \frac{3}{2}) \Gamma(2 + b - \alpha^g)}{\Gamma(\frac{3}{2}) \Gamma(3 + 2b - \alpha^g)}.
 \end{aligned}
 \tag{77}$$

Comparing this to (75) and (76) we conclude that both in the quark and gluon cases RDDA with $b = \alpha^{q,g}$ results in the very same skewness effect for small ξ as the minimalist dual model with the same input.

Thus, in the quark sector the minimalist dual model produces the non zero skewness effect for small ξ . It makes this model unpractical for the description of DVCS at small x_{Bj} due to systematical $\sim 50\%$ overshooting of the available H1 cross section data. At this point the minimalist dual model has no advantage comparing to the common version of RDDA plagued by the same problem (see [42] and the discussion in [13, 14]). At the same time, in the gluon sector the skewness effect in the minimalist dual model turns to be very small. It is worth to mention is the growing confidence that the gluon skewness ratio is also not given by the conformal ratio (76) (see discussion in [14]). The pure LO analysis of hard exclusive ρ^0 electroproduction rather hints that $r^g < 1$.

In fact one may recognize in (76) the well known result obtained in [38] using the Shuvaev integral transform approach [29]. In [14] it was suggested to call it as the conformal ratio

since this is nothing but a Clebsh-Gordan coefficient occurring in the conformal partial wave expansion. It is still unclear if the skewness effect (76) is an indispensable feature of GPD phenomenology (see discussion in [14, 38]).

Let us emphasize that the skewness ratio in the framework of the dual parametrization is not with necessity given by (76). In order to alter the skewness ratio one has to take into account the contribution of subsequent forward like functions $Q_{2\nu}$ ($G_{2\nu}$) singular enough to change the small ξ asymptotic behavior of $H^{qg}(\xi, \xi)$. For this one has to assume that $Q_{2\nu}(x)$, $G_{2\nu}(x) \sim 1/x^{2\nu+\alpha^{q,g}}$. The rigorous way to handle the occurring divergencies of the generalized form factors $B_{2\nu-10}$

$$\begin{aligned} B_{2\nu-10}^{q(E,M)}(t) &= \int_0^1 dx x^{2\nu-1} Q_{2\nu}^{(E,M)}(x, t); \\ B_{2\nu-10}^{g(E,M)}(t) &= \int_0^1 dx x^{2\nu-1} G_{2\nu}^{(E,M)}(x, t) \end{aligned} \quad (78)$$

and of the quark and gluon D -form factors was described in details in [22].

In the remaining part of this section we consider a toy model in order to briefly sketch the possible fitting strategy for hard exclusive scattering observables based on the dual parametrization of GPDs. For definiteness we discuss the case of DVCS at the leading order.

In order to fit the experimental data we have to propose certain Ansatz for the relevant GPD quintessence functions $N(x, t)$. The contribution of the forward like function $Q_0(x, t)$ into GPD quintessence function is entirely fixed in terms of the t -dependent parton distributions. Since for the moment we discuss DVCS let $q(x, t)$ be the DVCS combination of t -dependent singlet parton distributions $q(x, t) = \frac{4}{9}u_+(x, t) + \frac{1}{9}d_+(x, t)$. The observable quantities can be expressed in terms of the standard elementary amplitude

$$A(\xi, t) = \int_0^1 dx H(x, \xi, t) \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right], \quad (79)$$

where $H(x, \xi, t)$ is the DVCS combination of singlet quark GPDs.

The leading singular behavior for $x \sim 0$ of the t -dependent PDF $q(x, t)$ is assumed to be determined by the linear Regge trajectory $a(t) \equiv \alpha + \alpha't$:

$$q(x, t) \sim c^q \frac{1}{x^{a(t)}}, \quad (80)$$

where $c^q > 0$ is the numerical constant. The value of the intercept in the relevant kinematical domain is $\alpha(Q^2 = 4\text{GeV}^2) = 1.1 \div 1.2$, The slope parameter α' can be fixed *e.g.* with the help of the form factor sum rule [39]: $\alpha' = 1.1 \text{ GeV}^{-2}$.

The alluring possibility is to take advantage of the opportunities provided by the Abel transform tomography method and instead of modelling GPD quintessence function employ a model for the imaginary part of the elementary amplitude $\text{Im}A(\xi, t)$. The real part of the elementary amplitude $\text{Re}A(\xi, t)$ can be then computed with the help of GPD quintessence restored from $\text{Im}A(\xi, t)$ using the tomography method. The important advantage of this approach is that the amplitude computed in this way possess proper analytic properties in ξ and automatically satisfies the fixed t dispersion relation with one subtraction in the variable $\omega = \frac{1}{\xi}$ [33, 37]:

$$A(\xi, t) = 4D(t) + \frac{1}{\pi} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi' - i\epsilon} - \frac{1}{\xi + \xi' - i\epsilon} \right) \text{Im}A(\xi' - i\epsilon, t), \quad (81)$$

since the expression for $\text{Re}A(\xi, t)$ in the dual parametrization through the GPD quintessence function is equivalent to the dispersion relation (81).

Let us introduce the following notations: $\text{Im}A^{Q_0}$ and $\text{Im}A^{N-Q_0}$ for the contributions to the imaginary part originating from $Q_0(x, t)$ and $N(x, t) - Q_0(x, t)$ respectively. Analogously for the D form factor we introduce the notations D^{Q_0} and D^{N-Q_0} .

The leading singular behavior of $\text{Im}A^{Q_0}(\xi, t)$ computed from the $Q_0(x, t)$ corresponding to t -dependent PDF $q(x, t)$ with the asymptotic behavior (80) is

$$\text{Im}A^{Q_0}(\xi, t) \sim c^q \frac{2^{a(t)+1} \Gamma(\frac{1}{2}) \Gamma(a(t) + \frac{3}{2})}{\xi^{a(t)} \Gamma(a(t) + 2)} \equiv C^{Q_0}(t) \frac{1}{\xi^{a(t)}}, \quad (82)$$

where we define the function

$$\begin{aligned} C^{Q_0}(t) &\equiv c^q \frac{2^{a(t)+1} \Gamma(\frac{1}{2}) \Gamma(a(t) + \frac{3}{2})}{\Gamma(a(t) + 2)} \\ C^{Q_0}(0) &\simeq \frac{3\pi}{2} c^q \quad \text{for } \alpha \approx 1. \end{aligned} \quad (83)$$

The contribution of Q_0 to the D form factor can be computed using

$$D^{Q_0}(t) = \int_0^1 \frac{dx}{x} Q_0(x, t) \left(\frac{1}{\sqrt{1+x^2}} - 1 \right). \quad (84)$$

The skewness effect in the model which includes only the contribution of Q_0 is given by the conformal ratio (75). In order to make skewness effect satisfy H1 measurements [41] we

need to tune the small ξ asymptotic behavior of $\text{Im}A(\xi, t = 0)$. For this issue let us require the following asymptotic behavior of $\text{Im}A^{N-Q_0}$ for small ξ :

$$\text{Im}A^{N-Q_0}(\xi, t = 0) \sim -\frac{1}{3}C^{Q_0}(t = 0)\frac{1}{\xi^{a(0)}} \equiv C^{N-Q_0}(t = 0)\frac{1}{\xi^{a(0)}}. \quad (85)$$

This choice makes the skewness effect consistent with H1 results.

In order to parameterize the effect of the contribution of $N(x, t) - Q_0(x, t)$ to the imaginary part of the DVCS amplitude one may try to employ the following class of functions:

$$\text{Im}A^{N-Q_0}(\xi, t) = C^{N-Q_0}(t)\frac{1}{\xi^{a(t)}}(1 - \xi)^\beta. \quad (86)$$

The corresponding truly non-forward contribution to the GPD quintessence function $N(x, t) - Q_0(x, t)$ can be recovered from (86) employing the standard Abel transform tomography procedure and used to compute the corresponding contribution to the real part of the elementary amplitude $\text{Re}A^{N-Q_0}(\xi, t)$.

Finally, the value of the $N - Q_0$ contribution to the D form factor can be computed rule employing the analyticity assumptions with the help of inverse momentum sum rule [12, 13, 32]:

$$D^{N-Q_0}(t) = \int_{(0)}^1 \frac{d\xi}{\xi} \text{Im}A^{N-Q_0}(\xi, t) = C^{N-Q_0}(t)B(1 + \beta, -a(t)), \quad (87)$$

where B is the Euler beta function.

The analytical properties of $D^{N-Q_0}(t)$ in the variable t require much attention. The Euler beta function in (87) has poles in t for $t = \frac{-\alpha+k}{\alpha'}$ with $k = 0, 1, \dots$. There is a finite number of “tachion” poles at negative values of t which do not match with the t -channel resonance exchange picture forming the basis for the dual parametrization approach. *E.g.* for $1 < \alpha < 2$ there are two “tachion” poles at $t = -\frac{\alpha}{\alpha'}$ and $t = -\frac{\alpha-1}{\alpha'}$. This can be seen as an indication that the simple Regge motivated substitution is inadequate for modelling the t -dependence of the DVCS amplitude. A more sophisticated Ansatz with the non-trivial interplay between the ξ and t dependencies is needed for this issue.

The interesting possibility is to employ the general form (86) assuming the special form of the t -dependence of $C^{N-Q_0}(t)$ in order to get rid of “tachion” contributions into the D form factor:

$$C^{N-Q_0}(t) = C^{N-Q_0}(0) \left(t + \frac{\alpha}{\alpha'}\right) \left(t + \frac{\alpha-1}{\alpha'}\right) \frac{\alpha'^2}{\alpha(\alpha-1)}. \quad (88)$$

This results in the following expression for the D form factor:

$$D^{N-Q_0}(t) = C^{N-Q_0}(0) \frac{1}{\alpha(\alpha-1)} \frac{\Gamma(1+\beta)\Gamma(-a(t)+2)}{\Gamma(1+\beta-a(t))}. \quad (89)$$

The value of $C^{N-Q_0}(0)$ is necessarily negative in order to reduce the skewness effect for small ξ (see (85)). It is interesting to note that for $\beta > a(0) - 1$ the value of the D^{N-Q_0} form factor computed from (89) turns to be negative for $t = 0$ ⁵. It is explained in [3, 23, 43] that the negative value of the D form factor is intimately related to the spontaneous breaking of chiral symmetry in QCD. Some arguments in favor of the negative sign of the D form factor are also presented in [44] in the framework of the simple model of very large nucleus. In this case the first coefficient of the Gegenbauer expansion of the D form factor can be related to the surface tension of the hadron medium and its negative sign follows from the requirement of the mechanical stability of the system. Thus, at this point our model turns to be consistent with the achieved theoretical understanding of the underlying physical picture. Finally it is interesting to note that asymptotic behavior of the D form factor $D^{N-Q_0}(t)$ for $-t \rightarrow \infty$ is power like:

$$D^{N-Q_0}(t) \sim (-t)^{-\beta+1}. \quad (90)$$

This simple exercise shows the power of the analyticity assumptions allowing to unambiguity fix the subtraction constant in the dispersion relation (81) in terms of the absorptive part of the amplitude. The value of the D form factor turns to be determined by the small ξ asymptotic behavior of the DVCS cross section. It is extremely important to check whether this scenario is consistent with the available experimental data.

VII. CONCLUSIONS

In this paper we consider the application of the dual para-metrization approach to the case of gluon GPDs in a nucleon. We construct the partial wave expansion for both unpolarized and polarized gluon GPDs in the nucleon and present the explicit form of the integral transform allowing to rigorously sum up these formal series. We present the expressions for the elementary leading order amplitude entering the description of hard exclusive meson

⁵ One may check that the contribution of Q_0 (84) into the D - form factor is also a small negative number.

production in the GPD formalism. We also discuss the generalization of Abel transform tomography approach for the case of gluons. We argue that the dual parametrization provides opportunities for the more flexible modeling GPDs in a nucleon. We strongly suggest to use the fitting strategies based on the dual parametrization to extract the information on GPDs from the experimental data. Let us also stress that in principle there is no essential difference between the dual parametrization of GPDs and the Mellin-Barnes representation with the expansion of Gegenbauer moments in the in the t -channel $SO(3)$ [12, 14] (see discussion in [45]). However, the full inversion formula relating the two parametrizations is still unknown.

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A. SUMMING UP THE FORMAL SERIES FOR UNPOLARIZED GLUON GPDS

The method suggested in [15] for the summation of the formal partial wave expansions of the type (18) for GPDs in the framework of the dual parametrization consists in presenting GPD as the result of convolution of a certain convolution kernel with the set of forward like functions whose Mellin moment generate the generalized form factors B_{nl} .

The explicit form of the corresponding convolution kernel was presented in [15] for the case of quark GPD (see also [22] for the refined version of the derivation). In this appendix we present the summary of relations employed for the derivation of the integral transformation (28) expressing GPDs $H^{g(E)}$ through the set of the forward like functions $G_{2\nu}^{(E)}$.

We introduce the common useful variable

$$z_s = 2 \frac{z - \xi s}{(1 - s^2)y}, \quad (\text{A1})$$

with $0 < y < 1$ and consider the discontinuity⁶

$$\begin{aligned} \text{disc}_{z=x} \int_{-1}^1 ds \frac{1-s^2}{z_s^N} &= \frac{(-1)^{N-1}}{\Gamma(N)} \int_{-1}^1 ds (1-s^2) \delta^{(N-1)}(x_s) \\ &= (-1)^{N-1} \theta \left(1 - \frac{x^2}{\xi^2} \right) \frac{y^N}{2^N \xi^N \Gamma(N)} \left(\frac{\partial}{\partial s} \right)^{N-1} (1-s^2)^{N+1} \Big|_{s=\frac{x}{\xi}} \end{aligned} \quad (\text{A2})$$

Employing the Rodriguez formula for the Gegenbauer polynomials $C_n^{\frac{5}{2}}(z)$ [46]:

$$(1-z^2)^2 C_{n-1}^{\frac{5}{2}}(z) = \frac{(-1)^{n-1}}{2^n \Gamma(n)} \left(1 + \frac{5n}{6} + \frac{n^2}{6} \right) \left(\frac{\partial}{\partial z} \right)^{n-1} (1-z^2)^{n+1} \quad (\text{A3})$$

one can derive the following basic relation

$$\text{disc}_{z=x} \left(1 + y \frac{\partial}{\partial y} + \frac{1}{6} y^2 \frac{\partial^2}{\partial y^2} \right) \int_{-1}^1 ds \frac{1-s^2}{z_s^N} = \frac{y^N}{\xi^N} \theta \left(1 - \frac{x^2}{\xi^2} \right) \left(1 - \frac{x^2}{\xi^2} \right)^2 C_{N-1}^{\frac{5}{2}} \left(\frac{x}{\xi} \right). \quad (\text{A4})$$

Now being based on the result (A4) we introduce the function

$$F^{(2\nu)}(z, y) = \left(1 + y \frac{\partial}{\partial y} + \frac{1}{6} y^2 \frac{\partial^2}{\partial y^2} \right) \int_{-1}^1 ds \xi^{2\nu} z_s^{2-2\nu} \frac{1-s^2}{\sqrt{z_s^2 - 2z_s + \xi^2}} \quad (\text{A5})$$

whose discontinuity at $z = x$ reads

$$\begin{aligned} &\text{disc}_{z=x} F^{(2\nu)}(z, y) \\ &= \text{disc}_{z=x} \left(1 + y \frac{\partial}{\partial y} + \frac{1}{6} y^2 \frac{\partial^2}{\partial y^2} \right) \int_{-1}^1 ds \xi^{2\nu} \sum_{l=0}^{\infty} \xi^l z_s^{-2\nu-l+1} (1-s^2) P_l \left(\frac{1}{\xi} \right) \\ &= \theta \left(1 - \frac{x^2}{\xi^2} \right) \left(1 - \frac{x^2}{\xi^2} \right)^2 \sum_{l=0}^{\infty *} y^{2\nu+l-1} C_{2\nu+l-2}^{\frac{5}{2}} \left(\frac{x}{\xi} \right) \xi P_l \left(\frac{1}{\xi} \right). \end{aligned} \quad (\text{A6})$$

The asterisk in the sum in (A6) denotes that for $\nu = 0$ the terms with $l = 0$ and $l = 1$ are actually absent.

⁶ The discontinuity of the function $f(z)$ is defined as $\text{disc}_{z=x} f(z) = \frac{1}{2\pi i} (f(x-i0) - f(x+i0)) = \frac{1}{\pi} \text{Im} f(x-i0)$

Eq. (A6) is the natural building block for the desired convolution kernel. Indeed, according to the definition of the gluon forward like functions $G_{2\nu}^{(E)}(y, t)$ with $n = 2\nu + l - 1$

$$B_{n+1-2\nu}^{g(E)}(t) = \int_0^1 dy y^n G_{2\nu}^{(E)}(y, t) \quad \text{with} \quad n \geq 2\nu - 1, \quad \text{odd.} \quad (\text{A7})$$

Using (A7) together with (A6) it is straightforward to check that the integral convolution

$$\sum_{\nu=0}^{\infty} \int_0^1 dy \frac{1}{2} \{ \text{disc}_{z=x} F^{(2\nu)}(z, y) + \text{disc}_{z=-x} F^{(2\nu)}(z, y) \} G_{2\nu}^{(E)}(y, t) \quad (\text{A8})$$

results in the formal series (18).

The trick that allows to derive the expression for the convolution kernel consists in the explicit calculation of the discontinuity of $F^{(2\nu)}(z, y)$ at $z = x$ stemming from the cut at $1 - \sqrt{1 - \xi^2} < x_s < 1 + \sqrt{1 - \xi^2}$ and from possible poles at $z_s = 0$:

$$\begin{aligned} & \int_0^1 dy G_{2\nu}^{(E)}(y, t) \text{disc}_{z=x} F^{(2\nu)}(z, y) \\ &= \frac{\xi^{2\nu}}{\pi} \int_0^1 dy G_{2\nu}^{(E)}(y, t) \left(1 + y \frac{\partial}{\partial y} + \frac{1}{6} y^2 \frac{\partial^2}{\partial y^2} \right) \int_{-1}^1 ds \frac{x_s^{2-2\nu} (1 - s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \theta(2x_s - x_s^2 - \xi^2) \\ & - \theta \left(1 - \frac{x^2}{\xi^2} \right) \left(1 - \frac{x^2}{\xi^2} \right)^{2\nu-3} \sum_{l=0}^{2\nu-3} C_{2\nu-l-3}^{\frac{5}{2}} \left(\frac{x}{\xi} \right) \xi P_l \left(\frac{1}{\xi} \right) \int_0^1 dy y^{2\nu-l-2} G_{2\nu}^{(E)}(y, t). \end{aligned} \quad (\text{A9})$$

Finally, the expression for the gluon GPD $H^{g(E)}$ through the set of the forward like functions reads

$$\begin{aligned} H^{g(E)}(x, \xi, t) &= \sum_{\nu=0}^{\infty} \int_0^1 dy \frac{1}{2} \{ \text{disc}_{z=x} F^{(2\nu)}(z, y) + \text{disc}_{z=-x} F^{(2\nu)}(z, y) \} G_{2\nu}^{(E)}(y, t) \\ &= \sum_{\nu=0}^{\infty} \frac{\xi^{2\nu}}{2} [H^{g(E)(\nu)}(x, \xi, t) + H^{g(E)(\nu)}(-x, \xi, t)] \\ &+ \sum_{\nu=1}^{\infty} \theta \left(1 - \frac{x^2}{\xi^2} \right) \left(1 - \frac{x^2}{\xi^2} \right)^2 \xi C_{2\nu-2}^{\frac{5}{2}} \left(\frac{x}{\xi} \right) B_{2\nu-1,0}^{g(E)}(t), \end{aligned} \quad (\text{A10})$$

where the functions $H^{g(E)(\nu)}(x, \xi, t)$ defined for $-\xi \leq x \leq 1$ are given by the following

integral transformations:

$$\begin{aligned}
H^{g(E)(\nu)}(x, \xi, t) = & \\
& \theta(x > \xi) \frac{1}{\pi} \int_{y_0}^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2} \right) G_{2\nu}^{(E)}(y, t) \right] \int_{s_1}^{s_2} ds \frac{x_s^{2-2\nu} (1 - s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \\
& + \theta(|x| < \xi) \frac{1}{\pi} \int_0^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2} \right) G_{2\nu}^{(E)}(y, t) \right] \left\{ \int_{s_1}^{s_3} ds \frac{x_s^{2-2\nu} (1 - s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \right. \\
& \left. - \frac{\pi}{\xi^{2\nu}} \left(1 - \frac{x^2}{\xi^2} \right)^2 \times \sum_{l=-1}^{2\nu-3} C_{2\nu-l-3}^{\frac{5}{2}} \left(\frac{x}{\xi} \right) \xi P_l \left(\frac{1}{\xi} \right) \frac{6y^{2\nu-l-2}}{(2\nu-l)(2\nu-l+1)} \right\},
\end{aligned} \tag{A11}$$

with $P_{-n}(\chi) \equiv P_{n-1}(\chi)$. Note, that in (A11) we employ the standard notations adopted in [15, 22]. Namely, $x_s = 2 \frac{x-\xi s}{(1-s^2)y}$, s_i , ($i = 1, \dots, 4$) stand for the four roots of the equation $2x_s - x_s^2 - \xi^2 = 0$ given by the following expressions:

$$\begin{aligned}
s_1 &= \frac{1}{y} \left(\mu - \sqrt{(1 - xy)(1 + \mu^2) - (1 - y^2)} \right); \\
s_2 &= \frac{1}{y} \left(\mu + \sqrt{(1 - xy)(1 + \mu^2) - (1 - y^2)} \right); \\
s_3 &= \frac{1}{y} \left(\lambda - \sqrt{(1 - xy)(1 + \lambda^2) - (1 - y^2)} \right); \\
s_4 &= \frac{1}{y} \left(\lambda + \sqrt{(1 - xy)(1 + \lambda^2) - (1 - y^2)} \right),
\end{aligned} \tag{A12}$$

where

$$\mu = \frac{1 - \sqrt{1 - \xi^2}}{\xi}; \quad \lambda = \frac{1}{\mu}. \tag{A13}$$

y_0 and $\frac{1}{y_1}$ are the two solutions of the equation $s_1 = s_2$; while y_1 and $\frac{1}{y_0}$ are the two solutions of the equation $s_3 = s_4$;

$$y_0 = \frac{x(1 + \mu^2)}{2} + \sqrt{\frac{x^2(1 + \mu^2)^2}{4} - \mu^2}; \tag{A14}$$

$$y_1 = \frac{x(1 + \lambda^2)}{2} - \sqrt{\frac{x^2(1 + \lambda^2)^2}{4} - \lambda^2}. \tag{A15}$$

B. POLARIZED GLUON GPDS

As usual, to sum up the formal series for \tilde{H}^g (20) we introduce the set of polarized gluon forward like functions $\Delta G_{2\nu}(y, t)$ whose Mellin moments generate the generalized form factors $\tilde{B}_{nl}^g(t)$. with $n = 2\nu + l - 1$

$$\tilde{B}_{n, n+1-2\nu}^g(t) = \int_0^1 dy y^n \Delta G_{2\nu}(y, t) \quad \text{with } n \geq 2, \quad \text{even.} \quad (\text{B16})$$

The resulting expression for \tilde{H}^g through $\Delta G_{2\nu}(y, t)$ reads

$$\tilde{H}^g(x, \xi, t) = \sum_{\nu=0}^{\infty} \left(1 - x \frac{\partial}{\partial x} - \xi \frac{\partial}{\partial \xi} \right) \frac{\xi^{2\nu}}{2} \left[\tilde{H}^{g(\nu)}(x, \xi, t) - \tilde{H}^{g(\nu)}(-x, \xi, t) \right], \quad (\text{B17})$$

where $\tilde{H}^{g(\nu)}(x, \xi, t)$ defined for $-\xi < x < 1$ is given by

$$\begin{aligned} & \tilde{H}^{g(\nu)}(x, \xi, t) \\ &= \theta(x > \xi) \frac{1}{\pi} \int_{y_0}^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2} \right) \Delta G_{2\nu}(y, t) \right] \int_{s_1}^{s_2} ds \frac{x_s^{2-2\nu} (1 - s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \\ &+ \theta(|x| < \xi) \frac{1}{\pi} \int_0^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2} \right) \Delta G_{2\nu}(y, t) \right] \left\{ \int_{s_1}^{s_3} ds \frac{x_s^{2-2\nu} (1 - s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \right. \\ &\left. - \frac{\pi}{\xi^{2\nu}} \left(1 - \frac{x^2}{\xi^2} \right)^2 \sum_{l=-1}^{2\nu-3} C_{2\nu-l-3}^{\frac{5}{2}} \left(\frac{x}{\xi} \right) \xi P_l \left(\frac{1}{\xi} \right) \frac{6y^{2\nu-l-2}}{(2\nu-l)(2\nu-l+1)} \right\}, \end{aligned} \quad (\text{B18})$$

For even N the polynomiality condition (10) require

$$\begin{aligned} & \int_0^1 dx x^{N-1} \tilde{H}^g(x, \xi, t) = \sum_{\substack{k=0 \\ \text{even}}}^N \xi^k \tilde{h}_{N,k}^g(t) = \\ & \xi^N \sum_{\substack{n=2 \\ \text{even}}}^N \sum_{\substack{l=1 \\ \text{odd}}}^{n+1} \tilde{B}_{nl}^g(t) \xi P'_l \left(\frac{1}{\xi} \right) \frac{n(1+n)(2+n)(3+n) \Gamma(\frac{5}{2}) \Gamma(N)}{9 \cdot 2^N \Gamma(1 + \frac{-n+N}{2}) \Gamma(\frac{7}{2} + \frac{-2+n+N}{2})}. \end{aligned} \quad (\text{B19})$$

The corresponding set of coefficients $\tilde{h}_{N,k}^g(t)$ is expressed through the generalized form factors

$\tilde{B}_{nl}^g(t)$ as follows

$$\begin{aligned} \tilde{h}_{N,k}^g(t) &= \sum_{\substack{n=2 \\ \text{even}}}^N \sum_{\substack{l=1 \\ \text{odd}}}^{n+1} \tilde{B}_{nl}^g(t) (-1)^{\frac{k+l-N+1}{2}} \frac{(-1+k-N) \Gamma(\frac{2-k+l+N}{2})}{3 \cdot 2^{k+1} \Gamma(\frac{1+k+l-N}{2}) \Gamma(2-k+N)} \\ &\times \frac{n(1+n)(2+n)(3+n) \Gamma(N)}{\Gamma(\frac{2-n+N}{2}) \Gamma(\frac{5+n+N}{2})}. \end{aligned} \quad (\text{B20})$$

The expression for the polarized forward like function ΔG_0 through the t -dependent polarized gluon density $\Delta g(y, t)$ reads

$$\Delta G_0(x, t) = -\frac{9}{2} x^2 \int_x^1 \frac{dy}{y^3} \Delta g(y, t) + 3x \int_x^1 \frac{dy}{y^2} \Delta g(y, t) + \frac{3}{2} \int_x^1 \frac{dy}{y} \Delta g(y, t). \quad (\text{B21})$$

Finally, to sum up the formal series for $\tilde{H}^{g(PS)}$ (21) the set of polarized gluon forward like functions $\Delta G_{2\nu}^{(PS)}(y, t)$ whose Mellin moments generate the generalized form factors $\tilde{B}_{nl}^g(t)$ with $n = 2\nu + l$

$$\tilde{B}_{n-2\nu}^{g(PS)}(t) = \int_0^1 dy y^n \Delta G_{2\nu}^{PS}(y, t) \quad \text{with } n \geq 2, \quad \text{even}. \quad (\text{B22})$$

The resulting expression for \tilde{H}^g through $\Delta G_{2\nu}$ reads

$$\tilde{H}^{g(PS)}(x, \xi, t) = \sum_{\nu=0}^{\infty} \frac{\xi^{2\nu}}{2} \left[\tilde{H}^{g(PS)(\nu)}(x, \xi, t) - \tilde{H}^{g(PS)(\nu)}(-x, \xi, t) \right], \quad (\text{B23})$$

where $\tilde{H}^{g(PS)(\nu)}(x, \xi, t)$ defined for $-\xi < x < 1$ is given by

$$\begin{aligned} &\tilde{H}^{g(PS)(\nu)}(x, \xi, t) \\ &= \theta(x > \xi) \frac{1}{\pi} \int_{y_0}^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2} \right) \Delta G_{2\nu}^{(PS)}(y, t) \right] \int_{s_1}^{s_2} ds \frac{x_s^{1-2\nu} (1-s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \\ &+ \theta(|x| < \xi) \frac{1}{\pi} \int_0^1 dy \left[\frac{1}{3} \left(1 - y \frac{\partial}{\partial y} + \frac{1}{2} y^2 \frac{\partial^2}{\partial y^2} \right) \Delta G_{2\nu}^{(PS)}(y, t) \right] \left\{ \int_{s_1}^{s_3} ds \frac{x_s^{1-2\nu} (1-s^2)}{\sqrt{2x_s - x_s^2 - \xi^2}} \right. \\ &\left. - \frac{\pi}{\xi^{2\nu}} \left(1 - \frac{x^2}{\xi^2} \right)^2 \sum_{l=0}^{2\nu-2} C_{2\nu-l-2}^{\frac{5}{2}} \left(\frac{x}{\xi} \right) P_l \left(\frac{1}{\xi} \right) \frac{6y^{2\nu-l-1}}{(2\nu-l+1)(2\nu-l+2)} \right\}. \end{aligned} \quad (\text{B24})$$

For even N the polynomiality condition (10) require that

$$\begin{aligned} \int_0^1 dx x^{N-1} \tilde{H}^{g(Ps)}(x, \xi, t) &= \sum_{\substack{k=0 \\ \text{even}}}^N \xi^k \tilde{h}_{N,k}^{g(Ps)}(t) \\ &= \xi^N \sum_{\substack{n=2 \\ \text{even}}}^N \sum_{\substack{l=0 \\ \text{even}}}^n \tilde{B}_{nl}^{g(Ps)}(t) P_l \left(\frac{1}{\xi} \right) \frac{n(1+n)(2+n)(3+n) \Gamma(\frac{5}{2}) \Gamma(N)}{9 \cdot 2^N \Gamma(1 + \frac{-n+N}{2}) \Gamma(\frac{7}{2} + \frac{-2+n+N}{2})}. \end{aligned} \quad (\text{B25})$$

The coefficients $\tilde{h}_{N,k}^{g(Ps)}(t)$ at powers of ξ in (B25) are given by

$$\begin{aligned} \tilde{h}_{N,k}^{g(Ps)}(t) &= \sum_{\substack{n=2 \\ \text{even}}}^N \sum_{\substack{l=0 \\ \text{even}}}^n \tilde{B}_{nl}^{g(Ps)}(t) (-1)^{\frac{k+l-N}{2}} \frac{\Gamma(\frac{1-k+l+N}{2})}{3 \cdot 2^{k+2} \Gamma(\frac{2+k+l-N}{2}) \Gamma(1-k+N)} \\ &\times \frac{n(1+n)(2+n)(3+n) \Gamma(N)}{\Gamma(\frac{2-n+N}{2}) \Gamma(\frac{5+n+N}{2})}. \end{aligned} \quad (\text{B26})$$

The expression for the pseudoscalar forward like function $\Delta G_0^{(Ps)}$ through the pseudoscalar combination of t -dependent polarized gluon densities $g^{(Ps)}(y, t) \equiv \Delta g(y, t) + \tau \Delta e^g(y, t)$ reads

$$\Delta G_0^{(Ps)}(x, t) = \frac{45}{2} x^2 \int_x^1 \frac{dy}{y^3} \Delta g^{(Ps)}(y, t) - 9x \int_x^1 \frac{dy}{y^2} \Delta g^{(Ps)}(y, t) - \frac{3}{2} \int_x^1 \frac{dy}{y} \Delta g^{(Ps)}(y, t). \quad (\text{B27})$$

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